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MISSILE WARHEAD AND SPECIAL PROJECTS LABORATORY

ANALYTICAL SECTION

TECHNICAL MEMORANDUM
NO. 127A53

MEAN PRESENTED AREA OF SMALL STEEL FRAGMENTS MEASURED BY MEANS OF FREE FALL IN A VISCOUS FLUID

BY

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OCTOBER 1958

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ABSTRACT

The mean presented areas of small irregular shaped steel fragments, within and beyond the lower limit of the icosahedron gage, were measured by a new method. This technique, based upon the free fall of a fragment in a viscous fluid and the fragment weight, yields results comparable in accuracy to the icosahedron gage; an average deviation of 4.3% and a maximum deviation of 10% from true values were observed. For fragments below 30 grains, an empirical expression for presented area was determined, as follows:

$$\bar{A} = \frac{W}{K_1 \ V + K_2}$$

where

$\bar{A}$ = Mean presented area, taken in the manner of the current literature as ¼ of the total surface area (sq. cms.).

W = Weight of fragment (grams).

V = Terminal velocity of fragment (cms./sec).

K_1 & K_2 = Constants depending upon the fluid density, viscosity, and temperature. K_1 & K_2 = .0758 and .4700 for liquid methyl silicone fluid (Dow-Corning 200 Fluid, density 0.971 gms/cc^3 at 25° C, kinematic viscosity, 200 centistokes at 25° C, and test temperature of 25° C ± 2° C). The expression above holds for chunky fragments with no major concave contours.

ACKNOWLEDGEMENT

The idea for measurement of fragment presented area by fall in a viscous fluid was generated by Mr. B. Fairbanks of Technical Laboratory Services, FREL, Picatinny Arsenal, and is reported in Reference 6. The author wishes to express his appreciation to Mr. Fairbanks, and his associate, Mr. R. McCloud, for their assistance in connection with the work reported herein.

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CONCLUSIONS

For steel fragments below 30 grain weight, mean presented areas can be measured with simple equipment in a reasonable time. An average deviation of 4.3% and a maximum deviation of 10% was achieved in this measurement for chunky fragments with no major concave contours.

These results were obtained with 32 fragments including:

a. Random shaped steel shell fragments previously measured on the icosahedron gage at B. R. L. (13.5 to 32 grains).

b. Steel parallelopipeds, cylinders, and spheres (0.32 to 54 grains).

c. Brass cylinders (0.97 to 5.1 grains)

Data obtained indicated that the mean presented area (as defined below) of random shaped steel fragments under 30 grains subscribe to the following empirical expression:

$$\bar{A} = \frac{W}{K_1 V + K_2} \quad \text{Equation (1)}$$

where: $\bar{A}$ - Mean presented area or ¼ of the total surface area, in the manner of the current literature (sq. cms.).

W - fragment weight (grams).

V - Terminal velocity of fragment (cms./sec.).

K_1 & K_2 - .0758 and .4700 for liquid methyl silicone fluid (Dow-Corning 200 fluid, density, 0.971 gms/cm^3 at 25°C, kinematic viscosity, 200 centistokes at 25°C, and ambient temperature of 25°C ± 2°C).

INTRODUCTION

In the pursuance of weapons lethality forecasts, the mean presented area of a random shaped fragment in free flight appears as a significant determining value.

The expression for the drag force producing a negative acceleration on the fragment is as follows:

$$-ma = C_d A \rho V^2$$

where:

a = acceleration

m = mass

A = presented area

ρ = air density

V = velocity

C_d = coefficient of drag

The mean presented area ($\bar{A}$), can be shown to be equal to one-fourth of the surface area of the fragment. Hereafter, the mean presented area may be referred to simply as $\bar{A}$.

Before the advent of the icosahedron gage in 1953, the measurement of the $\bar{A}$ of a fragment was an extremely laborious lengthy task involving a great many readings. B.R.L. Report No. 877 states that the icosahedron gage could make a measurement in less than one hour with an overall instrument error of ± 5% and was limited to fragments above .023 sq. in. for $\bar{A}$. The method of this report makes possible the measurement of small fragments with no observed lower limit in $\bar{A}$ and in the comparatively short time of ten minutes or less.

Mr. Bernard Fairbanks in his Instrumentation Report No. TR-380-58/1 dated June 12, 1958, "Determining the Presented Area of a Fragment from its Terminal Velocity in a Grease Column", suggested the use of the terminal velocity in viscous silicone fluid and developed an empirical equation for $\overline{A}$ of spheres of different sizes.

The subject of this report extends the method of free fall through a fluid for the determination of mean presented area to shapes other than spheres, i.e., to other regular shapes and to the practical case of small random shaped fragments.

SUMMARY OF RESULTS

Various regular shaped fragments such as spheres, cylinders, and rectangular parallelopipeds along with irregular fragments were measured with the following results:

Fragment Material & Shape	No. of Fragments Tested	Method Used	Weight Range Grains	*$\bar{A}$ Range Sq. CMS	$\bar{A}$ Average % Deviation From True Values
Steel Spheres	4	Equation (1)	.84-54	.045-.70	5.5
Steel Rod	7	" (1)	.32- 6.2	.033-.24	3.8
Steel Square	7	" (1)	1.0 -18.5	.075-.55	5.4
Steel Shell Fragments	8	" (1)	13.5 -32.3	.48-1.48	4.6
Brass Cylinders	5	" (1)	.98- 5.2	.053-.21	2.6
Copper Sphere	1	" (1)	4.137	.117	1.4
Steel Spheres	4	" (3)	.84-54.	.045-.70	6.3
Steel Spheres	4	" (4)	.84-54.	.045-70	2.0
Aluminum Cylinders	5	" (5)	.41-2.0	.067-.232	0.4

The $\bar{A}$ for regular shaped fragments, calculated from the terminal velocity measurement and equation (1), were compared to the values given by ¼ the total surface area.

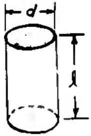
The $\bar{A}$ for the irregular shell fragments, calculated from the terminal velocity measurement and equation (1), were compared to the values received from Mr. M. Famiglietti of the Ballistic Research Laboratories.

An average deviation of 4.3% for 32 determinations was achieved in using equation (1).

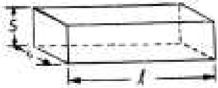
* $\bar{A}$ = Mean presented area

EXPERIMENTAL PROCEDURE

For the accumulation and analysis of data so that an empirical relationship could be found to measure $\bar{A}$, regular shaped steel cylinders and rectangular parallelopipeds were fabricated. These were made to the specification that the shape ratio of maximum regular to minimum regular dimension be within the limit of ½ to 5 and that the weights range below 25 grains. For cylinders, the shape ratio was taken as the length to diameter, while for square cross sectioned parallelopipeds, the shape ratio was taken as length to one side of the square of cross section as illustrated:



Shape ratio = $\frac{l}{d}$

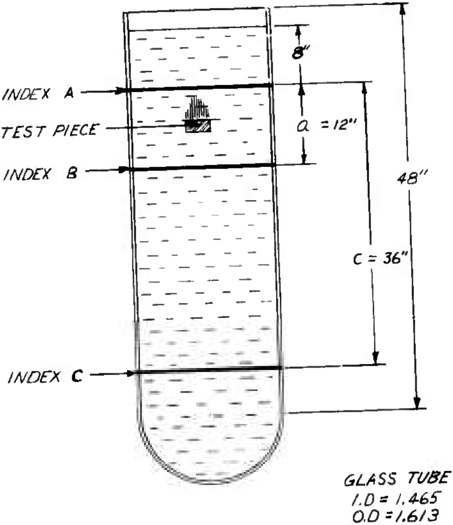


Shape ratio = $\frac{l}{s}$

These regular shaped pieces were dropped in Dow Corning 200 fluid, contained in the four foot glass tube as shown in Figure 1, and terminal velocity was calculated.

The Dow Corning 200 fluid used had a density of 0.971 and a kinematic viscosity of 200 centistokes at 25°C. The ambient temperature in the air conditioned room as 25°C ± 2°C.

The particle terminal velocity was calculated from the time of fall through 36". The time was the average of four individual falls. Orientation of the particle before drop did not affect the measured time and the particle



TEST CONTAINER SETUP FOR FRAGMENT TERMINAL VELOCITY IN A VISCOUS FLUID FIG. 1

always took the same preferred orientation in the fluid on reaching terminal velocity.

A test for terminal velocity was that the time of travel for the first 12" of 36" travel equal 1/3 of the total time for 36". This condition was met by allowing an 8 inch fall before initiating velocity timing as shown on sketch of glass tube.

Time was measured with a hand operated stop watch No. 144 made by Minerva of Switzerland, graduated in 1/10ths of a second. The watch was started and stopped at the indices signifying the beginning of the "a" interval and the end of the "c" interval as these indices were crossed by the falling fragment.

Before being dropped in the Dow Corning silicone fluid the particles were weighed with a chemical balance and measured for all significant dimensions with a micrometer.

After drawing various curves, the ratio of weight to mean presented area was plotted against terminal velocity, using the data for cylinders, rectangular parallelopipeds, and the data for spheres from Reference 6. A mathematical expression for mean presented area was derived from this graph. To test this expression, irregular shell fragments, previously measured by the Ballistic Research Laboratories, were remeasured using the method of this report. Likewise, spheres and cylinders of metals other than steel were used to check the validity of this method.

DISCUSSION

The evolution of a measurement method for obtaining mean presented area of an irregular shaped fragment has, by the very nature of the problem, involved a succession of lengthy and intricate approaches, each attempting to increase the reliability of the measurement and decrease the excessive time and labor required.*

The icosahedron gage as described in B.R.L. Report No. 877 represents the culmination of previous efforts and is the current instrument used. B.R.L. Report No. 877 states that measurement of a fragment can be made in less than an hour with a ± 5% overall instrument error, and the range of mean presented area measurable by the instrument is .023 $in.^2$ to 5 $in.^2$

B. Fairbanks, in his report on spheres dropped in a viscous fluid (Reference 6), took a distinctly different tack for the problem, arriving at an empirical expression. At the outset of the present investigation, an attempt was made to correlate the experimental terminal velocities for spheres from the report of B. Fairbanks with Stokes law for small spheres falling in a viscous fluid, namely: $V = \frac{2\,g\,r^2(d_1 - d_2)}{9n}$

where:

V = terminal velocity of a sphere falling in a fluid

g = acceleration of gravity

r = radius of the sphere

d_1 = density of small sphere

---

* Some of these approaches are presented in Ballistic Research Laboratories Report Nos. 877 and 501 listed in the "References".

d_2 = density of fluid

n = coefficient of viscosity of fluid

*The dimensions for spheres of different diameters, calculated from Stokes law, differed from the actual dimensions as shown in Table I and Figures Nos. 2 and 3. Since the radius squared was used in both the computing of actual sphere presented area and the Stokes equation, the correlation between actual radius and Stokes law calculated radius was based on the radius squared as follows:

1. Plotting r^2_{Stokes} against R^2 actual on logarithmic paper and achieving a straight line, the equation

$$R^2 \text{ actual} = 7.42\,(r^2_{Stokes})^{1.41} \qquad \text{Equation (3)}$$

was derived. The average deviation for $\bar{A}$ using equation (3) above was 6.3%.

2. A second correlation was worked out by inductive reasoning from the available data. Examination of the data pointed to an exponential relationship with a decreasing exponent as r^2_{Stokes} increased, resulting in the following expression:

$$R^2_{actual} = (r^2_{Stokes})^{\,1 - 4.7r^2_{Stokes}} \qquad \text{Equation (4)}$$

The average deviation for mean presented area using the R^2_{actual} from equation (4) was 2.0%.

Numerical data from equation (3) and (4) for various spheres and the resulting $\bar{A}$ deviations are to be found in Table I. Although equation (4) gave a small average deviation from the true value for spheres, correlation for other shapes could not be made and other avenues were sought.

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*See Appendix: Applicability of Stokes Law to the Measurement of Sphere Radius

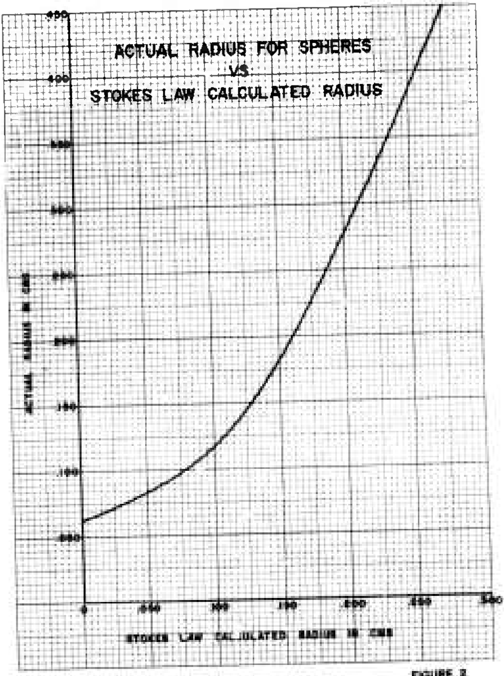

ACTUAL RADIUS FOR SPHERES
VS
STOKES LAW CALCULATED RADIUS
ACTUAL RADIUS IN CMS
.450
.400
.350
.300
.250
.200
.150
.100
.050
0
.050
.100
.150
.200
.250
.300
STOKES LAW CALCULATED RADIUS IN CMS

FIGURE 3

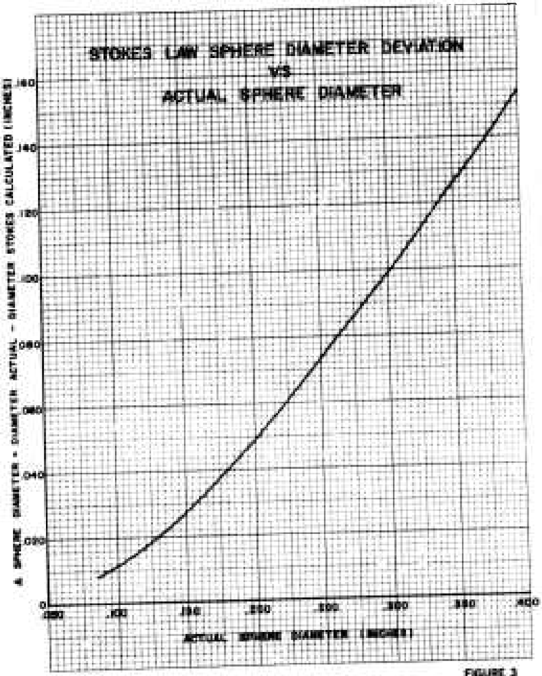

STOKES LAW SPHERE DIAMETER DEVIATION
VS
ACTUAL SPHERE DIAMETER
Δ SPHERE DIAMETER = DIAMETER ACTUAL − DIAMETER STOKES CALCULATED (INCHES)
.160
.140
.120
.100
.080
.060
.040
.020
0
.050
.100
.150
.200
.250
.300
.350
.400
ACTUAL SPHERE DIAMETER (INCHES)

FIGURE 3

TABLE 1

[illegible]

[illegible]

[illegible]	[illegible] inches	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Case III	[illegible]	[illegible]	[illegible]
Steel	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0449	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Steel	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Steel	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Steel	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
													[illegible]	[illegible]	[illegible]			[illegible]
Copper	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]							[illegible]				[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]							[illegible]				[illegible]	[illegible]	[illegible]

[illegible]

Terminal velocities for uniform circular and square cross sections of steel for increasing lengths (shape ratio: $\frac{1}{4}$ to 5) were plotted against actual $\bar{A}$ ($\frac{1}{4} \sum A$) of the particles and against their weights in Figures 4 and 5 respectively. The similarity of the latter two curves is quite striking, implying that weight and $\bar{A}$ hold a close relation to length in its effect on terminal velocity for a uniform cross section of varying length. This might be expected, since for a uniform cross section particle, weight is directly proportional to length and $\bar{A}$ is more directly a function of length as the lateral area becomes the dominant contributing factor for increasing lengths.

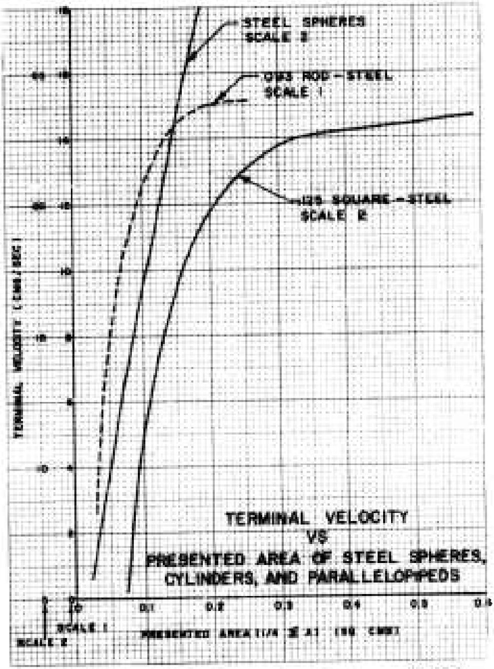
Plotting terminal velocity against shape ratio for both steel cylinders and parallelopipeds in Figure 6, note is made that terminal velocity increases sharply below a shape ratio of one, decreases sharply between the shaperatios of one to three, and that the terminal velocity approaches a constant for shape ratio beyond three.

In attempting to establish an empirical expression for $\bar{A}$ the following was assumed:

1. Weight and $\bar{A}$ were considered as opposing factors in their effect upon particle acceleration in the fluid.
2. Terminal velocity was considered proportional to weight.
3. Terminal velocity was considered inversely proportional to $\bar{A}$.
4. From the above considerations, the expression

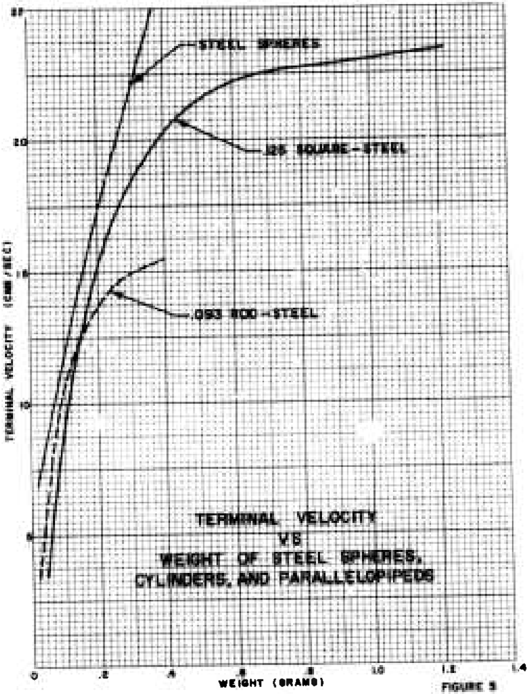
$$\text{Terminal Velocity} \propto \frac{\text{Weight}}{\bar{A}}$$

was explored, and the ratio, weight to $\bar{A}$, was solved as a function of terminal



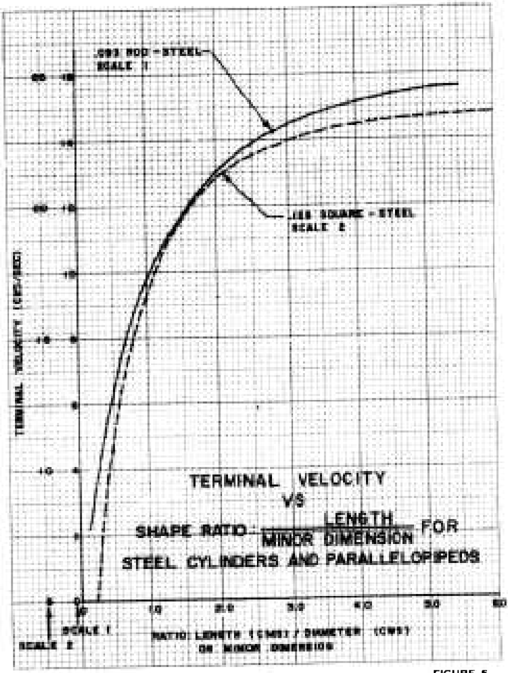
TERMINAL VELOCITY
VS
PRESENTED AREA OF STEEL SPHERES,
CYLINDERS, AND PARALLELOPIPEDS

FIGURE 4



TERMINAL VELOCITY
VS
WEIGHT OF STEEL SPHERES,
CYLINDERS, AND PARALLELOPIPEDS

FIGURE 5



.093 ROD - STEEL
SCALE 1
.188 SQUARE - STEEL
SCALE 2
TERMINAL VELOCITY (CMS/SEC)
TERMINAL VELOCITY
VS
SHAPE RATIO: LENGTH / MINOR DIMENSION FOR
STEEL CYLINDERS AND PARALLELOPIPEDS
0
1.0
2.0
3.0
4.0
5.0
6.0
SCALE 1
SCALE 2
RATIO: LENGTH (CMS) / DIAMETER (CMS)
OR MINOR DIMENSION

FIGURE 6

velocity, i.e., $\frac{\text{Weight}}{\bar{A}}$ = f (Terminal velocity) an expression to be tested empirically.

Plotting terminal velocity in cms/sec against the ratio of weight in grams to $\bar{A}$ in sq. cms. for the regular shaped circular and square cross sectioned steel particles in Figure No. 7, a straight line was produced with the following equation:

$$\frac{\text{Weight}}{\bar{A}} = .0758\ V + .4700 \qquad \text{Equation (1)}$$

$\bar{A}$ and $\bar{A}$ deviation % for small steel cylinders and small steel parallelopipeds were calculated using equation (1) and tabulated in Tables 2 and 3. The $\bar{A}$ average % deviations for the steel cylinders and parallelopipeds were 3.8% and 5.4% respectively.

For steel spheres, the actual ratio of weight to $\bar{A}$ was computed and plotted against terminal velocity on Figure No. 7. These plotted points for spheres distributed reasonably along the curve of Figure 7. The $\bar{A}$ and $\bar{A}$ % deviations for spheres were calculated using equation (1) and tabulated in Table 1. $\bar{A}$ average deviation % for spheres was 5.5%.

Steel shell fragments, previously measured for $\bar{A}$ by B.R.L., were dropped in the viscous fluid and their terminal velocities were computed. $\bar{A}$ for these fragments was calculated using equation (1) and tabulated in Table 4. The $\bar{A}$ average deviation % from the B.R.L values was 4.6%.

Uniform circular cross sectioned brass particles were tested for terminal velocity, and $\bar{A}$ was calculated using equation (1) and tabulated in Table 5. The $\bar{A}$ average deviation % was 2.6%.

Uniform circular cross sectioned aluminum particles were tested for terminal velocity and $\bar{A}$ was calculated using equation (1) and tabulated in

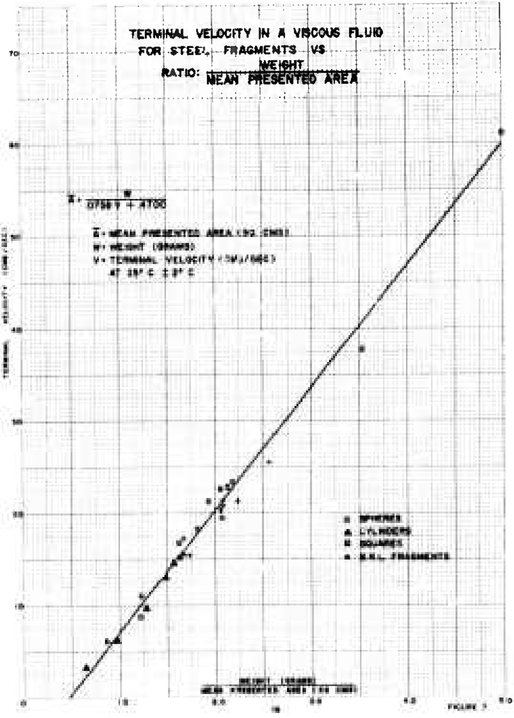

TERMINAL VELOCITY IN A VISCOUS FLUID
FOR STEEL FRAGMENTS VS
RATIO: WEIGHT / MEAN PRESENTED AREA
$\bar{A} = \frac{W}{.0758\,V + .4700}$
$\bar{A}$ = MEAN PRESENTED AREA (SQ. CMS)
W = WEIGHT (GRAMS)
V = TERMINAL VELOCITY (CMS/SEC)
AT 25° C ± 2° C
TERMINAL VELOCITY (CMS/SEC)
WEIGHT (GRAMS) / MEAN PRESENTED AREA (SQ CMS)
SPHERES
CYLINDERS
SQUARES
N.O.L. FRAGMENTS
70
60
50
40
30
20
10
0
1.0
2.0
3.0
4.0
5.0

FIGURE [illegible]

TABLE 2
Mean Presented Area of Small Cylinders of Steel

Measured by Terminal Velocity in [illegible] Viscous Fluid

Length/Dia. Ratio	Length inches	Length cms.	Weight grams	Weight grains	Velocity cms/sec	Surface Area (Σ_A) sq.cms.	Actual * $\bar{X}$ (ΣA) sq.cms.	Weight in gms / $\bar{X}$ in sq.cms Ratio — Actual	Weight in gms / $\bar{X}$ in sq.cms. Ratio — Computed from .0758V + .4700	Computed $\bar{X}$ sq.cms.	$\bar{X}$ Deviation sq.cms.	Deviation %
5.[illegible]2	.4[illegible]56	1.184	.4020	6.[illegible]	15.5	.[illegible]	.[illegible]42	1.66	1.[illegible]5	.244	+.[illegible]2	[illegible].8
4.00	.37[illegible]4	.946	.32[illegible]7	4.[illegible]	15.1	.[illegible]90	.198	1.63	1.32	.199	+.001	[illegible].5
3.01	.28[illegible]1	.712	.2432	3.75	14.5	.516	.154	1.57	1.57	.154	0	0
2.01	.1869	.475	.1637	2.52	13.[illegible]	.[illegible]	.110	1.4[illegible]	1.45	.113	+.003	+2.7
1.[illegible]0	.[illegible]935	.238	.[illegible]8	1.3[illegible]	[illegible].70	.204	.066	1.28	1.20	.[illegible]70	+.004	+[illegible].1
[illegible].50[illegible]	.0471	.112	.0420	.[illegible]50	[illegible].10	.171	.043	.975	.93	.045	-.00[illegible]	+4.[illegible]
0.[illegible]53	.[illegible]236	.060	.[illegible]210	.3[illegible]4	3.41	.132	.033	.634	.72	.029	-.004	-1[illegible].1
												Average Deviation % [illegible]

Additional Data:

Same diameter for all cylinders - .093 ins.

Velocity measured by timing [illegible] 3[illegible] in. distance

Viscous fluid - Dow Corning 20[illegible] fluid, 20[illegible] centistokes at 25°C

Ambient Temperature - 25° C ± 2°C

Velocity measured 11 July [illegible]

* $\bar{X}$ = Mean Presented Area

TABLE 3
Mean Presented Area of Small Square Cross Section Parallelopipeds

TABLE 3
Mean Presented Area of Small Square Cross Section Parallelopipeds of Steel Measured by Terminal Velocity in a Viscous Fluid

Length / one side of square Ratio	Length inches	Length cms.	Weight grams	Weight grains	Velocity cms/sec	Surface Area (Σa) sq.cms.	Actual * $\bar{X}$ ($\frac{1}{4}\Sigma a$)	Weight in gms. / $\bar{X}$ in sq.cms. Ratio - Actual	Weight in gms. / $\bar{X}$ in sq.cms. Ratio - Computed from .075 W + .4700	Computed $\bar{X}$ sq.cms.	$\bar{X}$ Deviation sq.cms.	Deviation %	
5.03	.6240	1.584	1.1946	18.5	23.4	2.193	.548	2.19	.34	[illegible]	-.012	-2.2	
4.03	.4990	1.268	.9544	14.7	22.9	1.794	[illegible]	2.13	.20	.433	-.015	-2.4	
3.03	.3760	.957	.7195	11.1	[illegible]	1.401	.350	2.05	2.17	.332	.017	-5.1	
2.02	.2510	.638	.4842	7.48	21.3	1.001	.250	1.93	.97	.233	-.017	[illegible]	
1.02	.1250	.320	.2417	3.74	16.9	[illegible]	.150	1.61	1.75	.138	-.012	-8.0	
0.513	.0630	.162	.1209	1.91	11.0	[illegible]	.101	[illegible]	1.30	.0954	[illegible]	-5.9	
.0258	.0320	.0815	.0643	.99	[illegible]	[illegible]	.075	.857	.92	.070	-.005	-6.7	
												Average Deviation % 5.[illegible]	

Additional Data:

Same square cross section for all parallelopipeds - .1439 ins. x .1233 ins.

Velocity measured by timing a [illegible] in. distance.

Viscous Fluid - Dow Corning 200 fluid, 2[illegible] centistokes at [illegible]

Ambient Temperature - 25°C ± 2°C

Velocity measured 11 July 1955

* $\bar{X}$ = Mean Presented Area

TABLE 4

Measurement of Mean Presented Area for Irregular Shell Fragments of Steel by Terminal Velocity in a Viscous Fluid and Comparison of Values with Previous Measurements at Ballistic Research Laboratories

Weight grams	Weight grains	Velocity cms sec	* $\bar{A}$ Measurement by BRL) sq.cms.	Weight grams / $\bar{A}$ sq.cms. (BRL values) Ratio Actual	Weight grams / $\bar{A}$ sq.cms. Ratio Computed from .0758 V - .4[illegible]	$\bar{A}$ computed sq.cms.	$\bar{A}$ Deviation sq.cms.	Dev. %
[illegible]	[illegible]	17.25	.0515	[illegible]	1.790	.6073	-.04[illegible]	- 5.[illegible]
1.4[illegible]	[illegible]	[illegible]	.8192	1.743	[illegible]	[illegible]	+.04[illegible]	+ 6.0
[illegible]	[illegible]	12.29	.4[illegible]37	1.[illegible]9	1.[illegible]50	.47[illegible]	-.0123	- [illegible]5
1.[illegible]532	[illegible]	[illegible]	.7998	[illegible]	2.[illegible]	[illegible]25	+.[illegible]	+ 2.[illegible]
[illegible]	[illegible]	[illegible]	.5[illegible]70	2.[illegible]	[illegible].082	.58[illegible]	-.0[illegible]4	- 0.0
[illegible]	[illegible]	[illegible].78	.45[illegible]	[illegible].08[illegible]	[illegible]45	.4671	+.00[illegible]1	+ 2.0
1.3526	[illegible]	[illegible]	[illegible]	2.[illegible]5	2.0[illegible]	.0497	+.049	+ .3
[illegible]	[illegible]	25.[illegible]5	.81[illegible]	2.570	[illegible].369	.8[illegible]37	-.0710	+ [illegible].7
								Av. Dev. % 4.[illegible]

* $\bar{A}$ = Mean Presented Area

[illegible]

[illegible] Fluid [illegible] [illegible], [illegible] centistokes at [illegible]

[illegible] temperature - [illegible]

TABLE 5

TABLE 5

[illegible] Area For [illegible] of Brass and Alum[illegible]

[illegible] Terminal Velocity in a Visc[illegible]us Fluid

Metal	Length	[illegible]	[illegible]	[illegible]	[illegible]	Velocity	Surface Area	C [illegible]	[illegible]	[illegible]	Computed X	Deviation	Deviation		[illegible]	Computed	Deviation
	[illegible] atic						[illegible]	(½Σ [illegible]) [illegible]	[illegible]	[illegible] Ratio computed from [illegible]	[illegible]	[illegible]	[illegible]		[illegible]	for aluminum	[illegible]
[illegible]	[illegible] [illegible] 3.17 [illegible] 61	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] 2.0				
Aluminum	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]

[illegible]

Table 5. As seen in Table 5, the $\overline{A}$ average deviation % was extremely high, 33.4% for these aluminum particles indicating that aluminum fragments do not follow the empirical constants in equation (1). The ratio of weight to actual $\overline{A}$ for the aluminum particles was re-computed and plotted against terminal velocity in Figure 8. The plot gave a straight line with the following equation:

$$\frac{\text{Weight}}{\overline{A}} = .0916\ V + .1538 \qquad \text{Equation (5)}$$

$\overline{A}$ was calculated using equation (5) and tabulated in Table 5. $\overline{A}$ average deviation % was 0.4%.

Note is made that reasonable results were achieved using the constants in equation (1) for steel, copper and brass. The densities of the latter three metals range from 7.6 to 8.9 gms/cm^3. However, for aluminum with a density of 2.69 gms/cm^3, a significant departure from the 7.6 - 8.9 density range, another set of constants, K_1 and K_2, had to be developed, as shown in equation (5).

In using the method of this report for measuring $\overline{A}$ of irregular fragments below 30 grains in weight, the advantages are as follows:

1. Short time per fragment measurement.
2. No lower limit for fragment weight or mean presented area.
3. No complex instrumentation.
4. No substantial instrument cost.
5. No involved techniques.
6. Low space requirement.

In actual operation, the weight to presented area ratio can be picked off an enlarged graph or table for any particular terminal velocity.

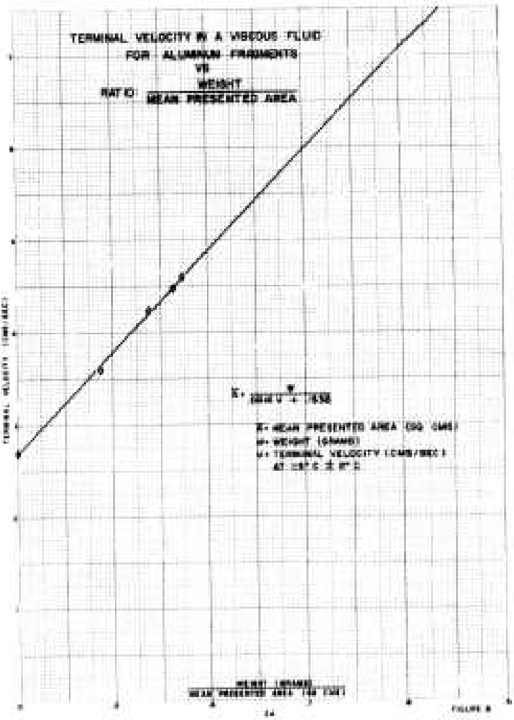

TERMINAL VELOCITY IN A VISCOUS FLUID
FOR ALUMINUM FRAGMENTS
VS
RATIO: WEIGHT / MEAN PRESENTED AREA
TERMINAL VELOCITY (CMS/SEC)
A = MEAN PRESENTED AREA (SQ. CMS)
W = WEIGHT (GRAMS)
U = TERMINAL VELOCITY (CMS/SEC)
AT 23° C ± [illegible]° C
WEIGHT (GRAMS) / MEAN PRESENTED AREA (SQ CMS)

FIGURE 8

APPENDIX

APPLICABILITY OF STOKES LAW TO THE MEASUREMENT OF SPHERE RADIUS

The resisting force on a sphere of radius r moving through a medium of viscosity η with constant velocity v was expressed by Stokes as follows:

$$F = 6\pi \eta r v \qquad (A)$$

For a free falling sphere, the resisting force is equal to the sphere weight minus the fluid buoyant force, and, if d_1 and d_2 are sphere and fluid densities respectively, equation (A) becomes

$$M_{(\text{sphere})}\, g - M_{(\text{displaced fluid})}\, g = 6\pi \eta r v$$

$$\tfrac{4}{3}\pi r^3 d_1 g - \tfrac{4}{3}\pi r^3 d_2 g = 6\pi \eta r v$$

$$v = \frac{\tfrac{4}{3}\pi r^3 g (d_1 - d_2)}{6\pi \eta r} = \frac{2 r^2 g (d_1 - d_2)}{9\eta}$$

$$r = \sqrt{\frac{9\eta v}{2g(d_1 - d_2}} \qquad (B)$$

Mr. B. Fairbanks, in his initial attempts at measuring mean presented area of spheres, tried to use the Stokes equation (B) and, getting no correlation, he developed the empirical expression reported in Instrumentation Report No. TR 380-58/1 in the "References."

The writer reported the results of using correction factors for Stokes law in the "Discussion" of this report and the deviation, between measured and Stokes law calculated sphere diameters versus actual sphere diameter, in Figure 3. From Figure 3, it can be seen that the deviation is approaching zero but is still appreciable for steel spheres below a weight of one grain as shown in the table below for some actual measurements:

Sphere Diam. in inches	.0937	.1562	.2500	.375
Sphere Stokes Diam Deviation %	10.7	19.3	29.3	37.0
Sphere Weight in grains	.8441	3.889	17.13	54.32

At this point note is made of comments by others on the applicability of Stokes law as follows:

1. Elementary Mechanics of Fluids by Hunter Rouse, 1946, Picatinny Library No. QA911 R

a. Page 158, middle of first paragraph: "Although its derivation is beyond the scope of this book, the expression for the longitudinal force F exerted by a slowly moving viscous fluid upon a small sphere, known as the equation of Stokes, is of interest at this point;

$$F = 3\pi D m v_0$$

This equation finds particular application to the fall of relatively small bodies through fluids of relatively high viscosity, - - -"

b. Page 244, first paragraph: "Resistance diagram for bodies of revolution. As indicated in Figure 125, a wealth of experimental data is at hand for the drag coefficient of spheres over a very great Reynolds-number range. At low values of R (i.e., in the zone of deformation drag) the measurements are seen to follow the straight line C = 24/R, which may be shown to correspond to the equation of Stokes Eq. (122) by the following operation:

$$F = 3\pi D m v_0 \times \frac{8 D \rho v_0}{8 D \rho v_0} = \frac{24 m}{v_0 D \rho} \frac{\pi D^2}{4} \frac{\rho v_0^2}{2} = \frac{24}{R} A \frac{\rho v_0^2}{2}$$

The experimental points begin to deviate from this line as soon as the accelerative effects, ignored by Stokes, begin to become appreciable; a Reynolds number slightly less than unity evidently marks the approximate limit of deformation drag, beyond which the Stokes equation is no longer applicable.

With increasing values of R the zone of appreciable viscous deformation becomes restricted more and more to the immediate boundary vicinity; at the same time, however, accelerative effects become more pronounced, and separation takes place in the zone of deceleration at the rear. By the time a Reynolds number of about 2 x 10^4 is reached, the viscous shear at the boundary has become so insignificant in comparison with the pressure reduction in the zone of discontinuity that the drag coefficient no longer varies perceptibly with the Reynolds number; this condition corresponds to the pressure distribution shown in Figure 120b."

Please note that for steel spheres used, Reynolds numbers ranged from 1.04 to 28.1.

2. Fluid Mechanics by Russel A. Dodge and Milton J. Thomson, 1937, Picatinny Library No. QA901 D6

a. Pages 175 & 176: "Experimental data on the resistance of spheres will be discussed in detail in Chap. XII, but at this point it may be mentioned that Stokes law holds only for a very restricted range of conditions. In the case of ordinary fluids, such as water and air, the size of the sphere must be so small as to be practically microscopic in character, while with larger spheres, either the fluid must be extremely viscous or the velocity must be very low. These latter cases are often referred to as 'creeping' motions.

In spite of these restrictions Stokes' law has not been without its practical applications. For example, it forms the basis for one method of measuring viscosity and has also been used to advantage in investigating the settling out of material suspended in liquids and in solving problems in diffusion."

b. Page 337: "Direct comparisons of the drag coefficients of bodies of revolution of different sections are difficult to make because of the variation of these coefficients with Reynolds' number. The most exhaustive studies in this connection have been made on spheres and the results of a large number of such investigations are shown in Figure 222. The extremely low values of N_r correspond to the viscous type of flow in which Stokes' law is valid. This solution as given by Eq. (11), $C_d = 24/n_r$, is also included in Figure 222, and it appears that this equation agrees with the experimental data only for values of N_r up to about 0.4. For higher values of N_r the inertia forces become more important and the drag coefficient decreases less rapidly with the Reynolds' number, approaching a practically constant value in the range from 10^3 to 10^5." (Figure 222 is reproduced on the next page).

In fluid flow around a sphere, there are inertia forces and viscous forces involved. For a low Reynolds number, the viscous forces are large compared to the inertia of the fluid particles. Stokes law solution for the drag of a sphere is based on this type of flow and breaks down when the inertia forces begin to predominate. This occurs for a Reynolds number of 0.4. Experimentally, small lead shot spheres, .0543" and .0385" diameter, were checked for terminal velocity in the Dow-Corning 200 silicone fluid at 25 ± 2°C. The resultant Stokes calculated diameter deviations were 0.9% and 2.5% - much smaller deviation percentages than for the previously mentioned larger steel balls.

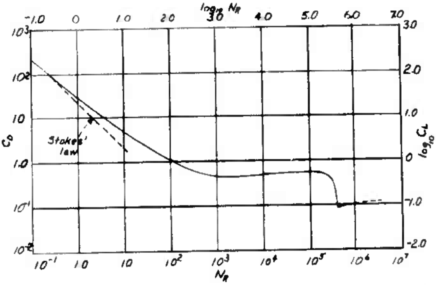


Fig. 222-Variation of drag coefficient of a sphere with Reynolds' number. (F. Eisner, Das Widerstandsproblem, Proc. Third Int. Cong. App. Mech. (Stockholm), 1931.)

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